other parts for different times at a fixed distance from the light. As a result it was found that, on development, the deposit was greatest when the exposure had been made at 2 ft. and diminished for each successive distance. By applying the measures of the different blacknesses obtained at the different distances to the curve obtained by the measurement of the scale of exposures, it was found that the exposure at 24 ft. ought to have been prolonged by 4·3 times to give the same blackness as that at 2 ft., the other distances giving intermediate results. If the law held good, the actual blackness of deposit at 24 ft. would have been obtained had the same exposure been given at about 50 ft. Other experiments are in progress, but it seemed advisable, without waiting for their completion, to make this addition to the paper, to show that the law fails both when short exposures and also feeble intensities of light are in question.

X. "On the Displacement of a Rigid Body in Space by Rotations. Preliminary Note." By J. J. WALKER, F.R.S. Received May 19, 1893.

Having been led to study more particularly than, as far as I am aware, has hitherto been done the conditions of the arbitrary displacement of a rigid body in space by means of rotations only, the results arrived at in the case of the single pairs of axes seem to me of sufficient interest and completeness to warrant their being recorded.

A comparison of these results with those arrived at by Rodrigues in his classic memoir "Des lois géométriques qui régissent les déplacements d'un système solide dans l'espace . . . ." 'Liouville,' vol. 5, 1840, at once suggesting itself, it may be proper here to recall the substance of the latter, and show how far they fall short of the object I propose to myself. The case of displacement by successive rotations round a pair of axes is discussed in § 13 (pp. 395—396), where it is shown that (p. 390), "Tout déplacement d'un système solide peut être représenté d'une infinité de manières par la succession de deux rotations de ce système autour de deux axes fixes non convergents. Le produit des sinus de ces demi-rotations multipliés par le sinus de l'angle de ces axes et par leur plus courte distance, est égal, pour tous ces *couples* d'axes *conjugués*, au produit du sinus de la demi-rotation du système autour de l'axe *central* du déplacement, multiplié par la demi-translation absolue du système."

Then (p. 396) the converse of this theorem is affirmed, viz., that "*Tout déplacement . . . peut toujours provenir, d'une infinité de manières, de la succession de deux rotations autours de deux axes non-convergents* pourvu que *le produit. . . .*"

In this conversion of the theorem above, it is strangely over-

looked that a displacement is not defined by the direction of axis, and amplitude, of the resultant rotation, together with the magnitude of the component of the corresponding translation along that direction (for in this form the proof is given, the axis being drawn through one end of the common perpendicular to the particular couple in respect of which the theorem is demonstrated), since these elements are common to an infinity of displacements.

This being premised, the laws connecting pairs of axes by successive rotations round which a given displacement of a rigid body in space may be effected are as follows:—

If the first axis (ζ' ξ') is taken arbitrarily, say parallel to a given vector, ζ', and passing through the term of a second given vector, ξ', its conjugate is parallel to a vector (ζ), the side common to three quadric cones, the constants of which are functions of ζ', ξ', and the vectors defining the displacement.

Each of these cones, whatever the direction of ζ', passes through one of three fixed vectors.

The directions of the axes being fixed in accordance with the above conditions, the locus of either axis is a plane, the places of the axes in which are so related that the connector of the feet of perpendiculars on them from any fixed point generates a ruled quadric surface.

[The last three paragraphs have been altered (July 15) after a correspondence, since the reading of the note on 15th June, with which Professor W. Burnside, F.R.S. (who, however, is not responsible for any statement herein), favoured me; as the result of which he sent me a geometrical proof that one axis might in all cases be taken arbitrarily both in position and direction. On revising my analysis, I found that what I had taken as an equation of condition was reducible to an identity.]

XI. "On a Graphical Representation of the Twenty-seven Lines on a Cubic Surface." By H. M. TAYLOR, M.A., Fellow of Trinity College, Cambridge. Communicated by A. R. FORSYTH, Sc.D., F.R.S. Received June 13, 1893.

(Abstract.)

The converse of Pascal's well-known theorem may be stated thus: if two triangles be in perspective, their non-corresponding sides intersect in six points lying on a conic. An extension of this theorem to three dimensions may be stated thus: if two tetrahedrons be in perspective, their non-corresponding faces intersect in twelve straight